Problem of the Month: *Measuring Up*

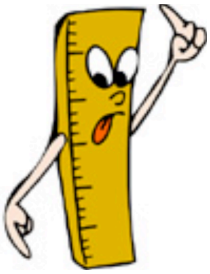
The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview:
In the Problem of the Month *Measuring Up,* students use algebraic thinking to solve problems involving proportional relationships, measurement, scale, and multiplicative relationships. The mathematical topics that underlie this POM are repeated addition, multiplication, division, percents, linear measurement, proportional reasoning, scale factors, scale, ratios, variables, functions, and algebraic reasoning.

In the first levels of the POM, students read a version of the story Stone Soup. In the story, a recipe for the soup is shared. The students are asked to determine how many carrots, onions, and chunks of meat are needed to feed various numbers of people. In level B, students are challenged with different proportional relationships between quantities in the Stone Soup recipe. They will need to use inverse relationships to determine some values. In level C, the students are presented with the challenge of determining a way to enlarge a picture to make a particular size poster. The copier only has single settings for enlarging and reducing. The students are asked to determine what combinations of enlarging and reducing are required to meet the poster size specifications. In level D, students analyze the relationship between two different measuring sticks that have different units. The students investigate when the units on the two sticks correspond. In the final level of the POM, students are presented with a situation that involves three broken rulers with differing measures. Students are asked to determine methods for converting between the three measuring sticks and formalizing their findings.

Mathematical Concepts:
The major mathematical ideas of this POM are measurement, proportional reasoning, and scaling. Students use multiplication, division, and representations of rational numbers such as fractions, decimal, and percents, as well as ratios, proportion, scale factors, similar figures, equations, and linear functions.



Problem of the Month
Measuring Up

Level A

A small group of six soldiers came into a small town. They were very hungry, but none of the townspeople offered them food. One of the soldiers announced that they would make *Stone Soup*. "How do you make Stone Soup a towns' person asked?" Well the soldier replied, "You need a big pot, water and a large stone." The townspeople, very curious to see how Stone Soup was made, gathered together the materials. The soldiers started to cook the soup over a fire they made. Once the soup began to boil, a soldier said, "sure this will be a tasty stone soup, but a delicious stone soup would have additional ingredients." The townspeople, now even more curious, asked what extra ingredients might be added. "Well for each person you would need 2 baby carrots, 3 green onions and five chunks of meat."

What ingredients are needed to make a delicious stone soup for the six soldiers?

What ingredients are needed to make a delicious stone soup for ten people?

What ingredients are needed to make a delicious stone soup for 25 people?

Explain how you determined you answers.

Level B

The townspeople brought more and more ingredients and put them in the soup. They began to lose track of how many people they could serve. One young girl who was careful to count the green onions announced that there were 69 green onions in the soup.

How many chunks of meat would need to be in the soup to make the recipe taste right?

How many people can be served soup with all these ingredients? Show how you figured it out.

One man said, "If we have 69 green onions, then I know we need 45 carrots." Is the man right, explain your answer.

Level C

You have designed a picture that is shaped as a square. The dimensions of your picture are 6 inches by 6 inches. You want to make it into a poster with dimensions 13 inches by 13 inches. The duplication machine has three settings, one that reduces the linear measure by a factor of 75%, one that enlarges linear measure by a factor of 140% and one that just makes identical copies or applies the factor of 100%. How many multiple enlargements or reductions will you need to make in order to create a poster that has its sides 13 inches long (accurate within five-hundredths)? Explain what setting was used and how that changed the measurements of the copy for each step in the process.

Level D

Two measuring sticks are the exact same length. The scale units on the two sticks are different. Each stick is marked with equally spaced units. The first stick starts at 0 and has 462 marks. The second stick starts at 0 and has 385 marks. Suppose the sticks were lined up so the two zero marks were matched. The 462 mark and the 385 mark would also be matched exactly. As you scan the measuring sticks, starting at zero, what is the very next set of marks on the two sticks that match exactly? What other marks match? Explain how you know.

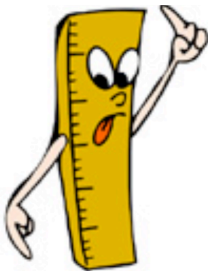
Level E

Arturo, Brennan, and Cameron each have broken measuring sticks. Each stick is marked with equally spaced units, but the units are not necessarily the same size from one stick to another. The first number appearing on Arturo's stick is 13. Brennan's stick starts with 32 and on Cameron's stick the number is 27.

They all held their sticks up next to the same chair and looked at the top number. Arturo's stick read 93, Brennan's stick read 92 and Cameron stick read 147.

Brennan measured Cameron's height using his stick in the same manner as he had the chair. It read 155. What reading would Arturo's measuring stick give for Cameron's height?

Determine a method for converting between the three different measuring sticks. If you measured something using Cameron's stick, what reading would you have on Arturo's and Brennan's sticks, and vice versa. Explain your solutions.



Problem of the Month
Measuring Up

Primary Version Level A

Materials: Sets of plastic food or color blocks.

Discussion on the rug: (Teacher asks the class.) "Who knows the story of Stone Soup?" (Teacher invites a student to tell about the story or she reads the story to the class). The teacher says to the host, "Suppose we wanted to feed two people stone soup. Let's start thinking how many vegetables we will need. " (The teacher invites students to share their ideas.) "How many carrots would we need?" the teacher asks the class. (Students share their answers and explain how they know.) The teacher says, "How many green onions and chunks of meat would we need?" (Students share their ideas. They may act it out to show the ingredient amounts. The teacher repeats the discussion for three people).

In small groups: (Students have counters available.)
Teacher says, "If you want to make stone soup for 10 people, how many carrots, green onions and chunks of meat do you need?" (Students work together to find a solution. After the students are done, the teacher asks students to share their answers and method.)

At the end of the investigation: (Students either discuss or dictate a response to this summary question.) "Explain and show how you know how many, each of the vegetable, you need for 10 people."

Problem of the Month

Measuring Up

Task Description – Level A

This task challenges a student to read a story with a given recipe. A student is to determine how many carrots, onions, and chunks of meat are needed to serve various numbers of people.

Common Core State Standards Math - Content Standards

Counting and Cardinality

Know number names and the count sequence.

K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Number and Operations in Base Ten

Use place value understanding and properties of operations to add and subtract.

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments

using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Problem of the Month

Measuring Up

Task Description – Level B

This task challenges a student to work with different proportional relationships between quantities in the recipe from Level A. A student will need to use inverse relationships to determine some values.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Represent and solve problems involving multiplication and division.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month

Measuring Up

Task Description – Level C

This task challenges a student to determine a way to enlarge a picture to make a particular sized poster. The copier only has single settings for enlarging and reducing. A student is asked to determine what combinations of enlarging and reducing are required to meet the poster size specifications. A student will need to think about and use proportional reasoning, measurement, and scale in this task.

Common Core State Standards Math - Content Standards

Number and Operations-Fractions

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.5 Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, …

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Common Core State Standards Math – Standards of Mathematical Practice

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students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Problem of the Month:

Measuring Up

Task Description – Level D

This task challenges a student to analyze the relationship between two measuring sticks that have different units. A student will investigate when the units on the sticks correspond to each other. Proportional relationships and reasoning will be used in this task.

Common Core State Standards Math - Content Standards

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Expressions and Equations 6.EE

Reason about and solve one-variable equations and inequalities.

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Solve real-life and mathematical problems using numerical algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Functions

Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

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MP.3 Construct viable arguments and critique the reasoning of others.

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established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Problem of the Month

Measuring Up

Task Description – Level E

This task challenges a student to determine methods for converting between three broken rulers with differing measures. The student must also formalize their findings.

Common Core State Standards Math - Content Standards

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Expressions and Equations

Reason about and solve one-variable equations and inequalities.

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Solve real-life and mathematical problems using numerical algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Functions

Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship …

High School – Algebra - Creating Equations

Create equations that describe numbers or relationships.

A-CED.2 Create equations in two or more variables to represent relationships between quantities; …

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

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Problem of the Month

Measuring Up

Task Description – Primary Level

This task challenges a student to understand the story of Stone Soup and the given recipe. Together, the class will decide how many carrots, green onions, and chunks of meat are needed for two people. In small groups, students will determine how many carrots, green onions, and chunks of meat are needed for 10 people. At the end of the investigation, a student will discuss or dictate a response to the summary question, "Explain and show how you know how much of each ingredient you will need for 10 people."

Common Core State Standards Math - Content Standards

Counting and Cardinality

Know number names and the count sequence.

K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Number and Operations in Base Ten

Use place value understanding and properties of operations to add and subtract.

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

Common Core State Standards Math – Standards of Mathematical Practice

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